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Quantum beats induced by an ultra-short excitation in a two-miniband superlattice

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Abstract
The coherent response of electrons in a double-well superlattice with two minibands when the superlattice is photoexcited with an ultra-short laser pulse is studied theoretically. Since the gap between the two minibands oscillates with the variation of the quasimomentum, not only do quantum beats—which depend on the parameters of the superlattice—occur, but also a non-dissipative quenching of the photoinduced current oscillations appears over time. These modifications of the oscillations are examined within the framework of the Kane model, using the parabolic approximation for electron and heavy-hole states and neglecting the Coulomb interaction. Numerical calculations of the carrier coherent dynamics are performed for the case of a δ-pulse excitation with a phenomenologically introduced damping factor.

1. Introduction

The ultra-fast photoexcitation of a physical system can generate quantum beats, which can be detected with four-wave mixing and THz emission experiments. In the latter technique the emitted electric field caused by the coherent motion of charges in a sample is detected. Thus, THz emission is directly related to the photoinduced current. Quantum beats are the double-quantum-well analogue of Bloch oscillations. The study of such phenomena in different semiconductor systems has attracted considerable attention during the last decade [1–4]. Tunnel-coupled heterostructures are among the most actively investigated objects in this area (see references [5] and [6] for a review)—including double quantum wells and superlattices subjected to transverse electric fields. Quantum beats are effectively excited in these structures due to the fact that the ultra-short pulse duration can be comparable to (or smaller than) the period of the coherent oscillations. This magnitude is determined by the level splitting energy.

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In the superlattice (SL) case, such level splitting appears when the structure is subjected to a transverse voltage (Wannier–Stark ladders). However, in the absence of an applied electric field, the level splitting in the tunnel-coupled wells is determined by the parameters of the structure. A way to raise the efficiency of quantum beats is to use a multi-double-well (MDW) structure. There is another alternative, little explored as yet: the double-well superlattice (DSL). A double-well superlattice can be formed from the MDW by reducing the width of the barrier that separates each set of two wells called the elementary cell (see figure 1(a)). Thus, the interlevel splitting energy becomes the gap between minibands (the interminiband gap). This gap oscillates with the variation of the quasimomentum (see figure 1(b)) and, therefore, the quantum beats become quenched.

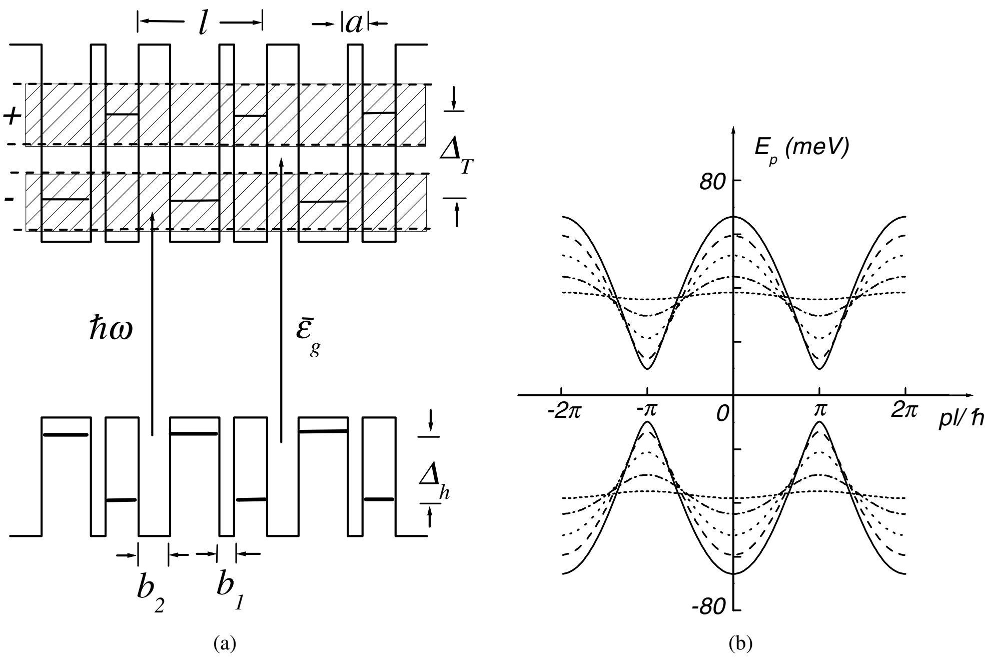

Figure 1. (a) The energy band diagram for a DSL with period l. (b) Electron dispersion laws for a two-miniband SL with different values of l. Solid line: $l = 96.3$ Å; dashed line: $l = 102.3$ Å; dotted line: $l = 112.3$ Å; dotted–dashed line: $l = 132.3$ Å; short-dashed line: $l = 162.3$ Å.

The aim of this paper is to study the coherent response of electrons, which are photoexcited with an ultra-short laser pulse, in DSL with different structural parameters. In this way we will consider a completely new character of the ultra-fast response in DSL. The qualitative peculiarities of the coherent response in DSL are associated with the above-mentioned oscillations of the gap. For this reason, the most important contribution to the quantum beats arises from the extremal (minima or maxima) values of the gap, which correspond to the centre and to the boundary of the one-dimensional Brillouin zone. Hence, the coherent response looks like beats of two oscillators. Contributions from another quasimomentum (i.e., from the spread of the interminiband gap over the Brillouin zone) lead to a *non-dissipative quenching* of these oscillations which appears to be proportional to the inverse square root of time.

Below, we present a simple analytical description of the transient evolution of the electronic density matrix in DSL after ultra-fast excitation. Using the tight-binding approach to describe electronic states and tunnel-uncoupled hole states, we treat interband optical transitions in the framework of the Kane model with parabolic energy spectra. Since the system under consideration is periodic along the DSL growth axis, we calculate the average current density using a single-particle description. We also restrict our study to the case of a δ-pulse excitation and use a phenomenological description of the relaxation [7]. As a result, we write the photoinduced current density in an explicit form, which takes into account the spread of harmonic oscillations with the variation of the interminiband frequency (corresponding to the interminiband gap) inside the first Brillouin zone of the DSL. These calculations are performed within a model of non-interacting electrons which leads to a substantial simplification of the calculations without losing the qualitative peculiarities of the response. The numerical analysis is carried out for non-symmetric DSL with different structural parameters. In particular, we use the actual non-symmetric DSL studied in reference [8] by means of electro-optical measurements. We also consider the limit case of a structure with identical quantum wells in each elementary cell, i.e., the usual SL with a period nearly half the DSL period.

The paper is organized as follows. In section 2 we present the basic equations for the ultra-fast coherent response. Section 3 includes the description of electron dynamics in a two-miniband DSL. Section 4 presents the analysis of quantum beats of the photoinduced current. The conclusions are given in the last section.

2. Coherent response

The coherent dynamics of electrons, when photoexcited by an ultra-short pulse, is described below in the framework of the second-order response of the interband excitation. Using a complete set of electron wave functions, Ψ_{qz}, to describe the electron motion along the DSL axis OZ, we write the photoinduced current J_{tz} in the form

$$J_{tz} = \frac{e}{2L^2} \sum_{pqq'} \left[(\hat{v}_z \Psi_{qz})^* \Psi_{q'z} + \Psi^*_{qz} \hat{v}_z \Psi_{q'z} \right] \rho_{q'q}(\boldsymbol{p}t). \tag{1}$$

Here L^2 is the normalization area, $\hat{v}_z$ is the velocity operator along the DSL axis and $\boldsymbol{p}$ is the 2D momentum. The average of the radiation density matrix over a period, $\rho_{qq'}(\boldsymbol{p}t)$, obeys the quantum kinetic equation (see equation (C.24) in reference [10] and reference [11])

$$\frac{\partial \rho_{q'q}(\boldsymbol{p}t)}{\partial t} + \frac{\mathrm{i}}{\hbar}(\varepsilon_{q'} - \varepsilon_q)\rho_{q'q}(\boldsymbol{p}t) = G_{q'q}(\boldsymbol{p}t) + I_{q'q}(\boldsymbol{p}t) \tag{2}$$

where $I_{q'q}(\boldsymbol{p}t)$ is the collision integral and $G_{q'q}(\boldsymbol{p}t)$ is the interband generation rate. For the near-edge excitation case with the in-plane electric field $E_\perp w_t \exp(-\mathrm{i}\omega t)$ + C.C., this generation rate takes the form

$$\begin{aligned} G_{q'q}(\boldsymbol{p}t) = \left(\frac{eE_\perp \mathcal{P}}{\hbar\omega}\right)^2 w_t \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega\tau} w_{t+\tau} \sum_{q_\mathrm{v}} \big[\phi(q', q_\mathrm{v}) \phi(q, q_\mathrm{v})^* \mathrm{e}^{\mathrm{i}(\varepsilon_{q'p} - \varepsilon_{q_\mathrm{v}p})\tau/\hbar} \\ + \phi(q_\mathrm{v}, q')^* \phi(q_\mathrm{v}, q) \mathrm{e}^{-\mathrm{i}(\varepsilon_{q_\mathrm{v}p} - \varepsilon_{qp})\tau/\hbar} \big] + \mathrm{H.C.} \end{aligned} \tag{3}$$

In equation (3) $\mathcal{P}$ is Kane's interband velocity, $E_\perp^2 = E_x^2 + E_y^2$, w_t is the envelope form factor of the laser pulse, with a frequency ω, and q, q' denote the electron states in the conduction (c) band while q_v corresponds to the valence (v) band states in DSL. The overlap integral in (3) is written as

$$\phi(q, q_\mathrm{v}) = \int \mathrm{d}z\, \Psi^*_{qz} \psi_{q_\mathrm{v}z} \tag{4}$$

where $\psi_{q_\mathrm{v}z}$ is the hole wave function.

For the approximation of parabolic dispersion relations, the energy of interband transitions in equation (3) is given by

$$\varepsilon_{qp} - \varepsilon_{q_v p} = \bar{\varepsilon}_g + \varepsilon_q - \varepsilon_{q_v} + \frac{p^2}{2\mu} \tag{5}$$

where $\bar{\varepsilon}_g$ is the effective gap between c and v states (which takes into account confinement effects in c and v bands), μ is the reduced mass, ε_q and ε_{q_v} are the energies of electron and hole states, respectively. Since the induced current is expressed through

$$\rho_{q'q}(t) = L^{-2} \sum_{\boldsymbol{p}} \rho_{q'q}(\boldsymbol{p}t)$$

equation (2) is transformed, after the summation over $\boldsymbol{p}$, to the following form:

$$\frac{\partial \rho_{q'q}(t)}{\partial t} + \frac{\mathrm{i}}{\hbar}(\varepsilon_{q'} - \varepsilon_q)\rho_{q'q}(t) = G_{q'q}(t) + J_{q'q}(t) \tag{6}$$

The above summation of equation (3) is performed with the use of the relation ('P' means the principal value)

$$\frac{1}{L^2} \sum_{\boldsymbol{p}} \exp[\mathrm{i}\tau p^2/(2\mu\hbar)] = \frac{\mu}{2\hbar}\delta(\tau) + \mathrm{i}\frac{\mu}{2\pi\hbar}\frac{\mathrm{P}}{\tau} \tag{7}$$

and the generation rate is transformed as follows:

$$G_{q'q}(t) = \frac{2N_{2\mathrm{D}}}{\tau_p} \sum_{q_v} \phi(q', q_v)\phi(q, q_v)^* \left\{ w_t^2 + \frac{\mathrm{i}w_t}{\pi} P \int_{-\infty}^{0} \frac{\mathrm{d}\tau}{\tau}\, w_{t+\tau} \mathrm{e}^{\mathrm{i}(\varepsilon_q - \varepsilon_{q_v})\tau/\hbar - \mathrm{i}\,\Delta\omega\,\tau} \right\} + \mathrm{H.C.} \tag{8}$$

Here we introduce the detuning frequency $\Delta\omega = \omega - \bar{\varepsilon}_g/\hbar$ and the characteristic 2D concentration:

$$N_{2\mathrm{D}} = \frac{\mu}{4\hbar} \left(\frac{eE_\perp \mathcal{P}}{\hbar\omega} \right)^2 \tau_p \tag{9}$$

where τ_p is the pulse duration in the Gaussian form factor $w_t = \exp[-(t/\tau_p)^2]$ used below.

The density matrix is expressed through $G_{qq'}(t)$, according to equation (6), in the form

$$\rho_{q'q}(t) = \int_{-\infty}^{t} \mathrm{d}\tau\; \mathrm{e}^{-\nu(t-\tau)} \mathrm{e}^{\mathrm{i}(\varepsilon_{q'} - \varepsilon_q)(t-\tau)/\hbar} G_{q'q}(t+\tau) \tag{10}$$

where the collision integral has been approximated by the interlevel relaxation frequency ν (below we only need the non-diagonal elements of the density matrix, $q \neq q'$). Thus, the photoinduced response is described by equation (1) after substitution of the above-obtained density matrix (10) with the generation rate (8).

3. Two-miniband dynamics

Hereafter, the eigenstate problem for electron and hole states is considered by using the tight-binding approach and assuming tunnel-uncoupled hole states. The extension of the electron wave function over the orbitals φ_z^{w} and φ_z^{n}, centred at wide (w) and narrow (n) wells forming the DSL, is given by

$$\Psi_z = \sum_k (\psi_k^{\mathrm{w}} \varphi_{z-kl}^{\mathrm{w}} + \psi_k^{\mathrm{n}} \varphi_{z-kl}^{\mathrm{n}}) \tag{11}$$

where $k = 0, \pm 1, \pm 2, \ldots$ is the position number of the elementary cells and l is the period of the DSL. The column wave function

$$\Psi = \begin{pmatrix} \ldots \\ \psi_k^{\mathrm{w}} \\ \psi_k^{\mathrm{n}} \\ \psi_{k+1}^{\mathrm{w}} \\ \psi_{k+1}^{\mathrm{n}} \\ \ldots \end{pmatrix} \tag{12}$$

is determined from the eigenstate problem $\hat{h}_{DSL}\Psi = E\Psi$ with the matrix Hamiltonian

$$\hat{h}_{DSL} = \begin{pmatrix} \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & -\Delta/2 & T_1 & 0 & 0 & \ldots \\ \ldots & T_1 & \Delta/2 & T_2 & 0 & \ldots \\ \ldots & 0 & T_2 & -\Delta/2 & T_1 & \ldots \\ \ldots & 0 & 0 & T_1 & \Delta/2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} \tag{13}$$

where Δ is the splitting energy between the levels of n and w quantum wells without tunnelling and $T_{1,2}$ stand for the different tunnel matrix elements (see figure 1). The matrix eigenstate problem is transformed into an infinite system of equations for the components of the column wave function (12):

$$\begin{aligned} T_2\psi_{k-1}^{\mathrm{n}} + T_1\psi_k^{\mathrm{n}} &= (E - \Delta/2)\psi_k^{\mathrm{w}} \\ T_1\psi_k^{\mathrm{w}} + T_2\psi_{k+1}^{\mathrm{w}} &= (E + \Delta/2)\psi_k^{\mathrm{n}}. \end{aligned} \tag{14}$$

We seek the solution of this system in the form $\psi_k^s = \phi_s \exp(\mathrm{i}kp_\perp l/\hbar)$, where $s = \mathrm{w}, \mathrm{n}$, and $p_\perp$ is the quasimomentum along the DSL, which is defined over the first Brillouin zone as $|p_\perp l/\hbar| < \pi$. The coefficients $\phi_{\mathrm{w,n}}$ are determined from coupled equations:

$$\begin{aligned} \{T_2 \exp(-\mathrm{i}p_\perp l/\hbar) + T_1\}\phi_{\mathrm{n}} &= (E - \Delta/2)\phi_{\mathrm{w}} \\ \{T_1 + T_2 \exp(\mathrm{i}p_\perp l/\hbar)\}\phi_{\mathrm{w}} &= (E + \Delta/2)\phi_{\mathrm{n}}. \end{aligned} \tag{15}$$

This eigenstate problem has a simple analytical solution, which allows us to write the dispersion laws for two minibands as follows:

$$E_{\pm p_\perp} = \pm\sqrt{(\Delta_T/2)^2 + 2T_1T_2\cos(p_\perp l/\hbar)} \tag{16}$$

where

$$\Delta_T/2 = \sqrt{(\Delta/2)^2 + T_1^2 + T_2^2}$$

is the mean interminiband gap. For the single-SL case ($\Delta = 0$ and $T_{1,2} = T$) we obtain a dispersion law $2T\cos(p_\perp l/2\hbar)$ in the reduced Brillouin zone $|p_\perp l/2\hbar| < \pi$. The wave functions for the eigenstate problem (15) take the forms

$$\frac{1}{\sqrt{2N_{SL}}}\begin{vmatrix} \phi_{\mathrm{w}}^{+p_\perp} \\ 1 \end{vmatrix} \qquad \frac{1}{\sqrt{2N_{SL}}}\begin{vmatrix} 1 \\ \phi_{\mathrm{n}}^{-p_\perp} \end{vmatrix} \tag{17}$$

with the components

$$\begin{aligned} \phi_{\mathrm{w}}^{+p_\perp} &= (E_{-p_\perp} + \Delta/2)/[T_1 + T_2\exp(\mathrm{i}p_\perp l/\hbar)] \\ \phi_{\mathrm{w}}^{-p_\perp} &= (E_{-p_\perp} - \Delta/2)/[T_1 + T_2\exp(-p_\perp l/\hbar)]. \end{aligned}$$

The normalization factor $1/\sqrt{2N_{SL}}$ appears here from the standard condition $\int \mathrm{d}z\ \Psi_z^*\Psi_z = 1$, and N_{SL} is the number of elementary cells along the DSL axis. Note that a similar dynamics was used for the description of the one-dimensional chains [12]; equations (11)–(17) provide the simple single-particle description of this problem.

The mixing of hole states caused by tunnelling is weak due to the large heavy-hole mass along the SL axis[4], so we use below a set of degenerate hole states. Each elementary cell, with two wells centred at $z = 0$ and $z = a$, respectively, contains two levels with energies 0 and $-\Delta_h$. These levels correspond to the hole orbitals $\varphi^{\rm w}_{z-rl}$ and $\varphi^{\rm n}_{z-rl}$ in the elementary cell, where $r = 0, \pm 1, \pm 2, \ldots$ is the position number of the cell. Thus we used here the hole quantum numbers $q_{\rm v} = (r, s)$ with $s = {\rm w, n}$.

The overlap factor in equation (3) is expressed through the above-introduced wave functions according to

$$\phi(\pm p_\perp, rs) = \int {\rm d}z\ \Psi^*_{\pm p_\perp z}\varphi^s_{z-rl} = \phi_s^{\pm p_\perp *}{\rm e}^{{\rm i}rp_\perp l/\hbar} \tag{18}$$

and the non-diagonal components of the velocity, which will appear in the photoinduced current (see equation (21) below), are written as

$$\begin{aligned} v_{jj'}(p_\perp) &= \frac{\rm i}{\hbar}(E_{jp_\perp} - E_{j'p_\perp})z_{jj'}(p_\perp) \\ z_{jj'}(p_\perp) &= \int {\rm d}z\ \Psi^*_{jp_\perp z} z \Psi^*_{j'p_\perp z} \qquad j \neq j'. \end{aligned} \tag{19}$$

Using the above wave functions we obtain the non-diagonal components of the coordinate in the form $z_{jj'}(p_\perp) \simeq aN_{SL}\phi_{\rm n}^{jp_\perp *}\phi_{\rm n}^{j'p_\perp}$.

4. Quantum beats

Furthermore, substituting the overlap factors (18) in (8) and performing the summation over r, we obtain a generation rate which is a diagonal function of quasimomenta:

$$\begin{aligned} G_{j'j}(p_\perp t) = \frac{2N_{\rm 2D}}{\tau_p}\Bigg\{ & w_t^2\delta_{jj} + \frac{{\rm i}w_t}{\pi}P\int_{-\infty}^0 \frac{{\rm d}\tau}{\tau}\ w_{t+\tau}{\rm e}^{{\rm i}E_{jp_\perp}\tau/\hbar - {\rm i}\,\Delta\omega\,\tau}N_{SL} \\ & \times \left[\phi_{\rm n}^{jp_\perp *}\phi_{\rm n}^{jp_\perp}{\rm e}^{{\rm i}\Delta_h\tau/\hbar} + \phi_{\rm w}^{jp_\perp *}\phi_{\rm w}^{jp_\perp}\right]\Bigg\} + {\rm H.C.} \end{aligned} \tag{20}$$

The first addendum in (20) has also been transformed using the orthogonalization condition

$$\sum_{s={\rm w,n}} \phi_s^{jp_\perp *}\phi_s^{j'p_\perp} = \delta_{jj'}.$$

Note that the non-diagonal contribution to the second addendum only appears when the condition $\Delta_h \neq 0$ is satisfied. Using the transformation of the wave function $\Psi_{jp_\perp z+l} = \exp(p_\perp l/\hbar)\Psi_{jp_\perp z}$ in equation (1), we prove the periodicity condition $J_{tz+l} = J_{tz}$, so we can consider below the photoinduced current averaged over the DSL period l:

$$J_t = \frac{1}{l}\int_{(l)} {\rm d}z\ J_{tz} = \frac{e}{l}\sum_{jj'} v_{jj'}(p_\perp)\rho_{j'j}(p_\perp t). \tag{21}$$

Since the diagonal component of the velocity is odd with respect to $p_\perp$ (i.e., $v_{jj}(-p_\perp) = -v_{jj}(p_\perp)$) while $\rho_{jj}(p_\perp t)$ is even (see equations (10) and (20)), only the interminiband contributions to J_t are essential. This result reflects the general point that the second-order response only appears in the non-symmetric system. Since the two-miniband electron states

[4] The in-plane hole mass is of the order of the electron effective mass (see reference [7]), so the reduced mass μ in equation (9) is small enough.

are invariant with respect to the replacement $z \longleftrightarrow -z$, the non-zero response occurs due to the difference of the hole states forming the elementary cell, if $\Delta_h \neq 0$.

Below we consider the δ-pulse excitation case, where the condition

$$\max|\Delta_T/\hbar,\, \Delta\omega|\, \tau_p \ll 1$$

holds. Under such an assumption, the non-diagonal (with $j \neq j'$) contribution to the generation rate (20) can be written in the simple form

$$G_{j'j}(p_\perp t) \simeq \frac{4N_{2\mathrm{D}}\Delta_h}{\pi\hbar} N_{SL}\phi_{\mathrm{n}}^{j'p_\perp *}\phi_{\mathrm{n}}^{jp_\perp}\overline{w}_t \tag{22}$$

where we have also introduced the effective form factor $\overline{w}_t = w_t \int_{-\infty}^{0} \mathrm{d}\tau\ w_{t+\tau}/\tau_p$. After introducing this generation rate in equation (21) and integrating over the quasimomentum (here we use the standard relation $\sum_{p_\perp} \cdots \to (l/\hbar)\int_{-\pi\hbar/l}^{\pi\hbar/l} \mathrm{d}p_\perp\ \cdots)$ we obtain the induced current as follows:

$$J_t \simeq \frac{\mathrm{i}ea}{\hbar l} \sum_{j\,(\neq j')} \int_{-\pi}^{\pi} \mathrm{d}\left(\frac{p_\perp l}{\hbar}\right) (E_{jp_\perp} - E_{j'p_\perp})|N_{SL}\phi_{\mathrm{n}}^{jp_\perp *}\phi_{\mathrm{n}}^{j'p_\perp}|^2$$
$$\times \frac{4N_{2\mathrm{D}}\Delta_h}{\pi\hbar\tau_p} \int_{-\infty}^{t} \mathrm{d}\tau\ \overline{w}_\tau \mathrm{e}^{-\nu(t-\tau)} \exp[-\mathrm{i}(E_{jp_\perp} - E_{j'p_\perp})(t-\tau)/\hbar]. \tag{23}$$

Performing the summation over j in equation (23) and using the δ-like dependency of $\overline{w}_\tau$, we finally transform J_t into the simple one-dimensional integral

$$J_t \simeq W_t \mathrm{e}^{-\nu t} \frac{2eN_{2\mathrm{D}}\Delta_h a}{\pi\hbar^2 l} \int_{-\pi}^{\pi} \mathrm{d}\alpha\ \xi_\alpha \frac{\xi_\alpha - \Delta}{\xi_\alpha + \Delta} \sin(\xi_\alpha t/\hbar) \tag{24}$$

with the dimensionless step-like function $W_t = \int_{-\infty}^{t} \mathrm{d}\tau\ \overline{w}_\tau/\tau_p$ and the interminiband energy

$$\xi_\alpha = 2\sqrt{(\Delta_T/2)^2 + 2T_1T_2\cos\alpha}.$$

Thus, the character of the quantum beats of J_t in (24) is determined by the dimensionless time-dependent function:

$$F(t) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\alpha\ \xi_\alpha}{\sqrt{2T_1T_2}} \frac{\xi_\alpha - \Delta}{\xi_\alpha + \Delta} \sin(\xi_\alpha t/\hbar) \tag{25}$$

where $\sqrt{2T_1T_2}$ characterizes the interminiband gap dispersion. This integral describes the spread of the harmonic oscillations with the interminiband frequency over the Brillouin zone of the DSL.

For a large-scale time, the integral (26) is estimated by using the method of the stationary phase. The main contributions to (25) appear due to the extrema of ξ_α localized at $|\alpha| = 0, \pi$. Introducing the maximal (+) and the minimal (−) energies

$$\xi_\pm = 2\sqrt{(\Delta_T/2)^2 \pm 2T_1T_2}$$

and using the condition $\xi_\pm t/\hbar \gg 1$, we obtain

$$F(t) \simeq \sum_{(\pm)} A_\pm \sqrt{\frac{\pi\hbar}{\xi_\pm t}} \sin\left(\frac{\xi_\pm t}{\hbar} \mp \frac{\pi}{4}\right) \qquad A_\pm = \frac{\xi_\pm^2}{4T_1T_2} \frac{\xi_\pm - \Delta}{\xi_\pm + \Delta}. \tag{26}$$

Thus, $F(t)$ is quenched with time as $t^{-1/2}$ and the double-frequency coherent response gives rise to long-period quantum beats. For the case of symmetric spectra of the electrons (when $\Delta = 0$ and $T_1 = T_2 \equiv T$), equation (26) becomes $F(t) \simeq 4\sqrt{\pi\hbar/Tt}\sin(4Tt/\hbar - \pi/4)$, so quantum beats have a period of the order of $T/\hbar$. Meanwhile, the second-order response remains non-zero (due to non-symmetry of hole states, $\Delta_h \neq 0$).

In the discussion of the coherent response, we consider the function (25) without the modulation factor $W_t \exp(-\nu t)$. The non-symmetry of the DSL is included in (25) through the parameters

$$\delta = \Delta/\sqrt{2T_1T_2} \qquad \eta = \sqrt{(T_1^2 + T_2^2)/2T_1T_2}$$

which describe contributions from the level splitting and from the different barrier penetrations. Figure 2 represents $F(t)$ versus dimensionless time $t\sqrt{2T_1T_2}/\hbar$ for the case $\delta = 0$, $\eta = 1$, showing, superimposed, the temporal behaviour of the non-dissipative damping ($\propto t^{-1/2}$) of oscillations. These values of δ and η correspond to a single SL with equal barriers and equal wells. As expected, quantum beats caused by the interminiband frequency do not appear for the single SL because there is only one miniband.

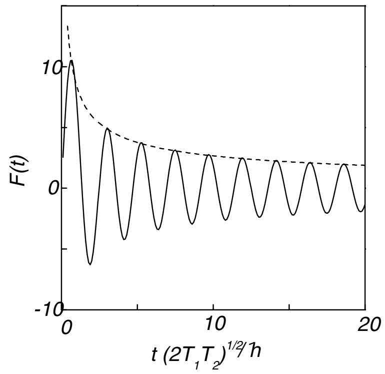

Figure 2. The dimensionless form factor $F(t)$ for $\delta = 0$, $\eta = 1$ (single SL). The dashed line represents the non-dissipative relaxation detailed in equation (26).

Quantum beats occur for DSL ($\delta \neq 0$), with a period that increases when increasing the decoupling between elementary cells (increase of η). Figure 3(a) shows the evolution of $F(t)$ for a fixed δ (we chose $\delta = 1$) and for different values of η ($\eta = 1, 2, 3, 4$) in four panels. A larger η means a larger difference between the widths of the two different kinds of barrier (b_1, b_2 in figure 1(a)). Thus we can see that the amplitude drastically increases as η increases. The frequency of coherent oscillation also increases with η. Figure 3(b) represents the other case: a fixed η-value ($\eta = 2$) and four different values of δ ($\delta = 0.5, 2, 4, 6$) in four panels. The larger δ, the more asymmetric the elementary cell and the wider the gap between minibands. Thus, we can see that the oscillation amplitude decreases when δ increases. Oscillation frequency increases and quantum beat frequency slightly decreases as δ increases.

In order to show the character of the response for a real structure, we perform calculations for the experimental DSL of reference [6]. Each elementary cell of this DSL is formed by two GaAl wells of width 36.8 Å and 31.1 Å, respectively, separated by a $Ga_{0.65}Al_{0.35}As$ barrier (b_1) of width 14.2 Å. A cell is separated from the adjacent cell by a barrier whose width (b_2) can be varied to analyse the evolution from the DSL to the MDW structure. In order to study the transition from a single SL to a DSL, we have also analysed a single SL with a 36.8 Å well width and a 14.2 Å barrier width, respectively. Actually, these superlattices are strongly coupled structures due to the barrier sizes. Figure 4 shows the evolution of the function $K(t) = W_t \exp(-\nu t)F(t)$ for the DSL versus time and for different values of b_2. In calculations we have used a relaxation frequency [13] $\nu = 0.3\ \text{ps}^{-1}$ and an excitation pulse duration $\tau_p = 10$ fs. In particular, we have represented in panel (a) the case $b_2 = 14.2$ Å ($=b_1$), in which the wells have the strongest coupling and the minibands are the broadest. It

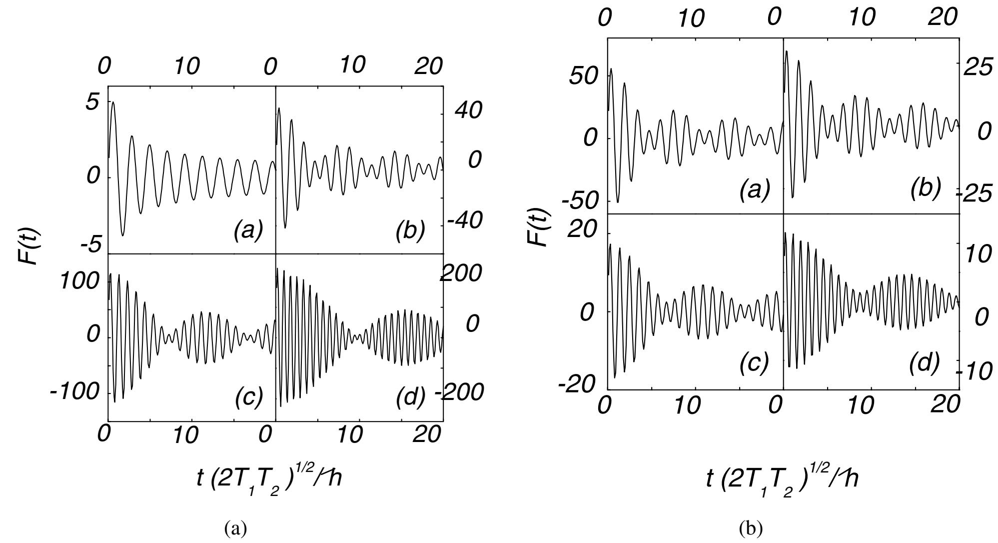

Figure 3. (a) The dimensionless form factor $F(t)$ for $\delta = 1$. Panel (a): $\eta = 1$. Panel (b): $\eta = 2$. Panel (c): $\eta = 3$. Panel (d): $\eta = 4$. (b) $F(t)$ for $\eta = 2$. Panel (a): $\delta = 0.5$. Panel (b): $\delta = 2$. Panel (c): $\delta = 4$. Panel (d): $\delta = 6$.

should be underlined that quantum beats are not very evident in this case because the oscillation period and the quantum beat period are very similar. Panel (b) corresponds to $b_2 = 30$ Å, panel (c) corresponds to $b_2 = 50$ Å and panel (d) to $b_2 = 80$ Å. For $b_2 \gtrsim 100$ Å quantum beats disappear due to the decoupling between elementary cells. Beyond these values we have a multiple-double-quantum-well system.

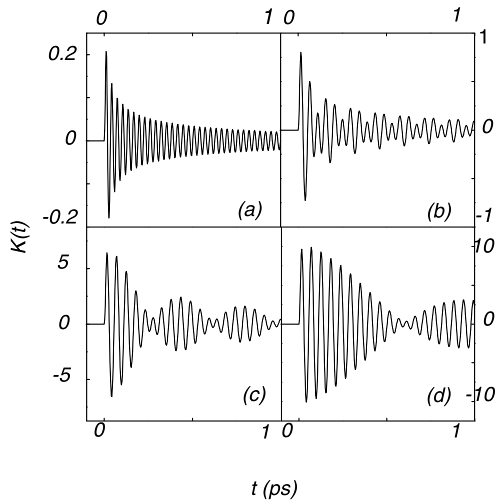

Figure 4. $K(t)$ for the experimental DSL described in the text. Panel (a): $b_2 = 14.2$ Å. Panel (b): $b_2 = 30$ Å. Panel (c): $b_2 = 50$ Å. Panel (d): $b_2 = 80$ Å. The curves correspond to the values $\nu = 0.3$ ps^{-1} and $\tau_p = 10$ fs.

5. Concluding remarks

This paper has shown the peculiarities of the coherent response of photoexcited electrons in a double-well superlattice with two minibands. The quantum beats after a δ-pulse excitation show a substantial dependency on the miniband spectrum parameters, which are determined by the interwell tunnel coupling. The comparison to the case of a multi-double-quantum-well structure allows us to find optimal DSL parameters for the effective excitation of quantum beats in the THz spectral region. Incidentally, we would like to emphasize that the theoretical consideration presented may be useful (after some modifications) for the interpretation of the recent experimental data from the ultra-fast spectroscopy of sodium atoms placed in a standing wave of light [14].

The single-particle approximation used in this work leads to substantial simplifications of the calculations. It is believed that the qualitative features of the response are still present if the Coulomb interaction is included and if the tunnel coupling of the holes is taken to be non-zero. This expectation should be substantiated by future more realistic calculations. In principle, in order to neglect Coulomb effects we have assumed that the mean energy of photoexcited electrons is large in comparison to the Bohr energy ε_B, i.e., $\tau_p < \hbar/\varepsilon_B$. In this case the general behaviour of quantum beats and the THz emission are determined by the electronic oscillations [15]. Otherwise, one has to use a direct numerical consideration in line with the results described in references [15–17]. As is stated in reference [9] (see also references therein), excitonic effects are essential for more sensitive non-linear effects, e.g., ultra-fast four-wave mixing. Meier *et al* [15] analyse in detail excitonic effects in the single SL, whereas Bott *et al* [16] place particular emphasis on a DSL, wider than the ones studied in our work.

To conclude, we have described the modification of the coherent dynamics of electrons, photoexcited by an ultra-short laser pulse, due to the two-miniband energy spectra in a DSL. Corresponding measurements enable us not only to verify these peculiarities, but also to select the most effective conditions for THz emission as well. It would also be interesting to study the effect of a transverse electric field when the complex structure of Wannier–Stark ladders in DSL can substantially change the quantum beats [2].

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